

# **Nuclear Anapole Moments and Their Constraints on the PNC Nuclear Interaction**

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- **What is the anapole moment?**
- **The PNC nuclear interaction and nuclear AMs**
- **Experimental and theoretical works**
- **Summary**

## ● What is the anapole moment?

■ From classical EM:

$$\Phi(\vec{x}) = \int d^3 x' \frac{\rho(\vec{x}')}{4\pi|\vec{x}-\vec{x}'|}$$

$$= \int d^3 x' \rho(\vec{x}') \left\{ 1 - \vec{x}' \cdot \vec{\nabla} + \frac{1}{2} (\vec{x}' \cdot \vec{\nabla})^2 + \dots \right\} \frac{1}{4\pi|\vec{x}|}$$

→ net charge, charge dipole, charge quadrupole ⊕ charge radius, ...

$$\vec{A}(\vec{x}) = \int d^3 x' \frac{\vec{j}(\vec{x}')}{4\pi|\vec{x}-\vec{x}'|}$$

$$= \int d^3 x' \vec{j}(\vec{x}') \left\{ 1 - \vec{x}' \cdot \vec{\nabla} + \frac{1}{2} (\vec{x}' \cdot \vec{\nabla})^2 + \dots \right\} \frac{1}{4\pi|\vec{x}|}$$

→ no net current, magnetic dipole, magnetic quadrupole ⊕ anapole, ...

■ Anapole Moment:

$$\vec{a} = -\frac{1}{4} \int d^3 x' \vec{x}'^2 \vec{j}(\vec{x}')$$

$$\vec{A}(\vec{x}) = \vec{a} \delta^3(\vec{x})$$

→  $\vec{a}$  is odd (even) for a vector (axial-vector) current

→  $\vec{A}(\vec{x})$  is of contact form

## ■ Multipole Classification:

	$C_J$	$E_J$	$M_J$
$J = 0$	PT		
$J = 1$	■	■T	PT
$J = 2$	PT	P■	■
$J = 3$	■	■T	PT
$\vdots$	$\vdots$	$\vdots$	$\vdots$

→  $C_0$  : charge monopole

→  $C_1$  : charge dipole

→  $M_1$  : magnetic dipole

→  $E_1$  : anapole

## ■ Definition:

→ The EM vector multipole which is P-odd but T-even

→ By multipole expansion:

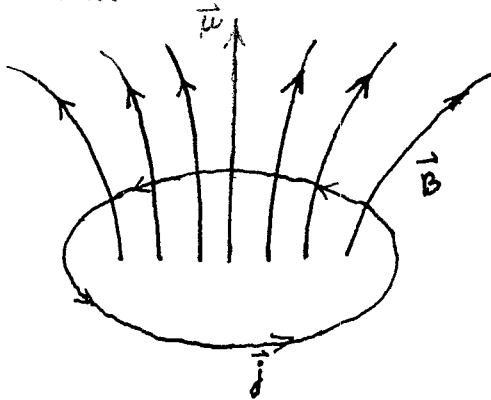
$$a_\lambda = \frac{-i\sqrt{6\pi}}{\hat{q}^2} T_{1\lambda}^{\text{el}}$$

→ A dimensionless constant  $\kappa$  is often used:

$$\hat{a} = \frac{G_F}{\sqrt{2}} \kappa \hat{I} \text{ (nuclear spin)}$$

● Look at the current windings

■ Circular



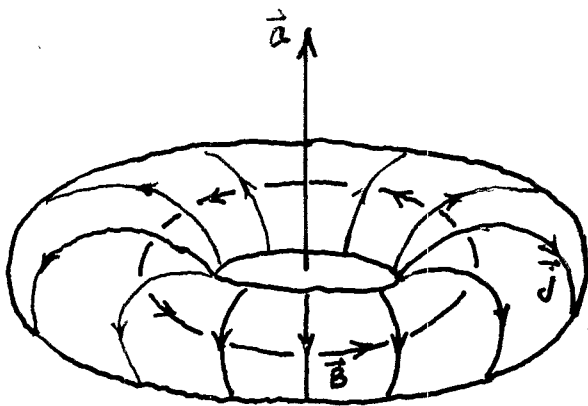
$\vec{j}(\vec{x})$  : toroidal

$\vec{B}(\vec{x})$  : poloidal (extensive)

$\vec{\mu} \neq 0$

$\vec{a} = 0$

■ Toroidal

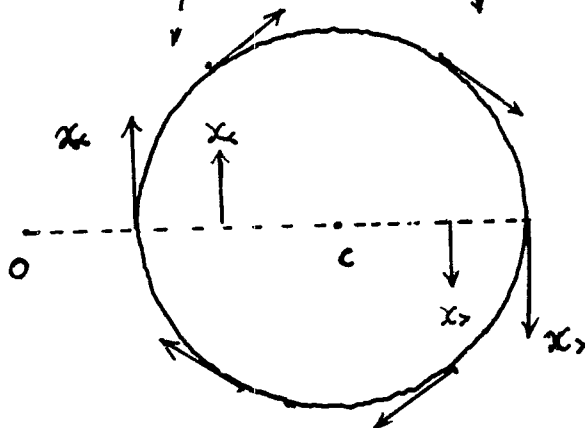


$\vec{j}(\vec{x})$  : poloidal

$\vec{B}(\vec{x})$  : toroidal (confined)

$\vec{\mu} = 0$

$\vec{a} \neq 0$



## ● The PNC nuclear interaction and nuclear AMs

**PNC nuclear interaction is important to understand both the hadronic neutral weak interaction and dynamics of strong interaction**

■ Long-range PNC N-N interaction from the light meson exchange scheme

→ Direct  $W^\pm$  and  $Z^0$  exchanges is suppressed by the hard core ( $\lesssim 0.5$  fm)

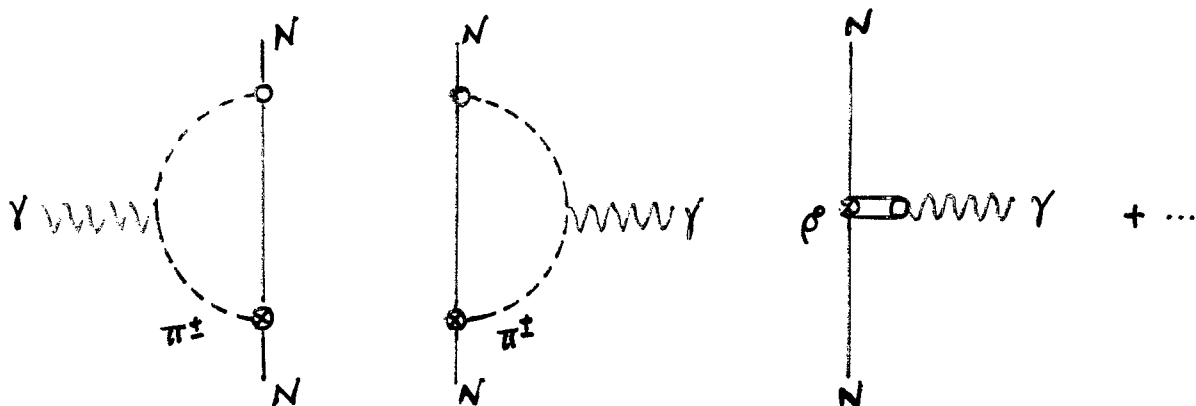
→ The physics of how weak bosons exchange between quarks at short distances is treated phenomenologically by a set of meson-nucleon PV couplings

→ Conventional framework is OBEP with  $\pi^\pm$ ,  $\rho$ , and  $\omega$  as mediators, the six PNC meson-nucleon coupling constants are constrained by experiments while DDH best values provide the theoretical benchmark

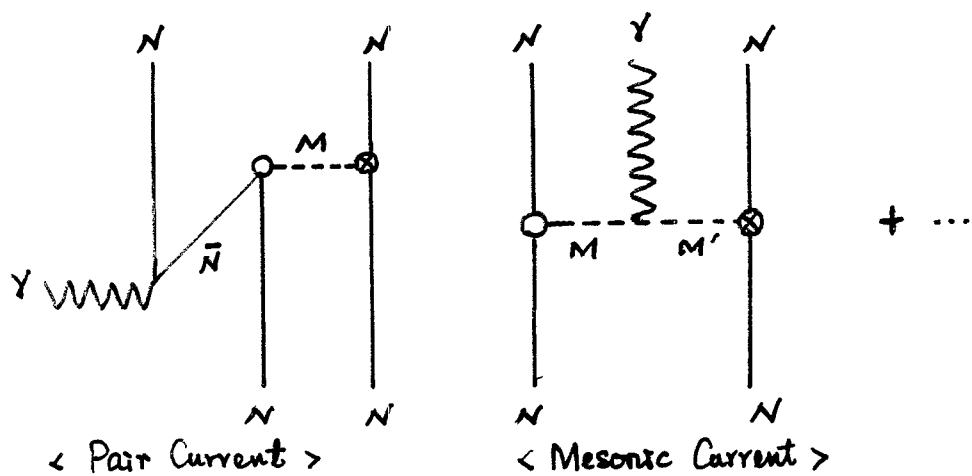
Couplings	Equivalent	Best ( $\times 10^{-7}$ )	Range ( $\times 10^{-7}$ )
$f_\pi$	$\sqrt{32} F_\pi / g_{\pi NN}$	4.56	0 ~ 11.4
$h_\rho^0$	$-2 F_0 / g_{\rho NN}$	-11.4	-30.78 ~ 11.4
$h_\rho^1$	$-2 F_1 / g_{\rho NN}$	-0.19	-0.38 ~ 0
$h_\rho^2$	$-2 F_2 / g_{\rho NN}$	-9.5	-11.02 ~ -7.6
$h_\omega^0$	$-2 G_0 / g_{\omega NN}$	-1.9	-10.26 ~ 5.7
$h_\omega^1$	$-2 G_1 / g_{\omega NN}$	-1.14	-1.9 ~ -0.76

■ PNC electromagnetic couplings induced by  $H_{\text{PNC}}^{(2)}$

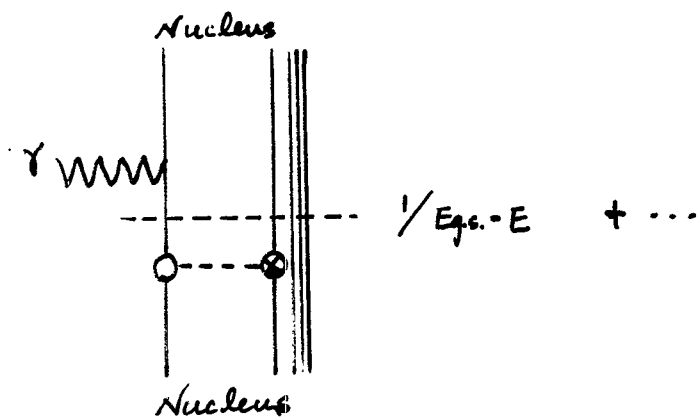
→ **1-body contribution** (nucleonic AM)



→ **2-body contribution** (PNC exchange currents)



→ **Polarization mixing** (E1 excitations)



## ● Experimental and theoretical works

### ■ Atomic PNC experiments

→ Asymmetry in highly forbidden M1 transitions

e.g.  $6S_{1/2} \longleftrightarrow 7S_{1/2}$  in  $^{133}\text{Cs}$

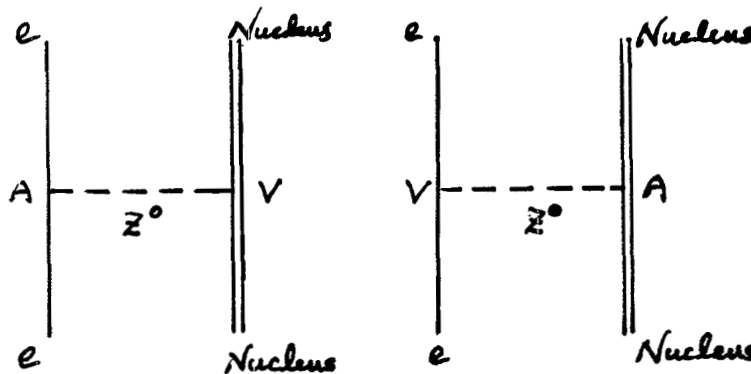
→ Optical rotation in allowed M1 transitions

e.g.  $6P_{1/2} \longleftrightarrow 6P_{3/2}$  in  $^{203,205}\text{Tl}$

### ■ Contributing processes

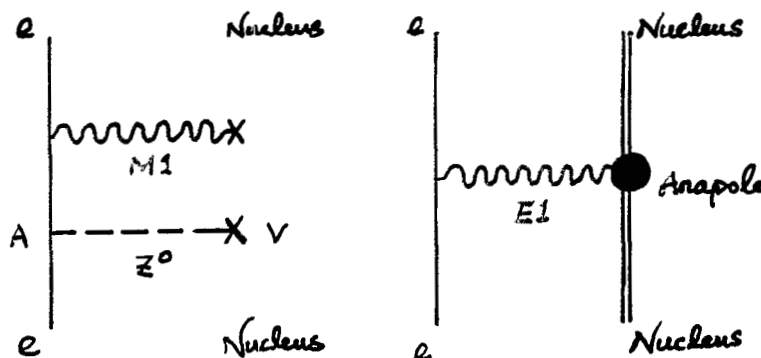
#### (1) $Z^0$ exchange

→ NSI part ( $\propto Q_W$ ) dominates, NSD part suppressed by  $1-4\sin^2\theta_W$



#### (2) NSD radiative corrections of $O(G_F \alpha)$

→ AM gives the dominant NSD effect for heavy nuclei ( $\sim A^{2/3}$ )



< Hyperline Correction >

< Anapole Interaction >

## ■ Extraction of anapole contribution

$$H_{\text{PNC}}^{\text{NSD}}(e - \text{nucleus}) = \frac{G_F}{2} \kappa_{\text{tot}} \vec{\alpha} \cdot \vec{I} \rho(\vec{r})$$

$$\kappa_{\text{tot}} = \kappa_Z + \kappa_{\text{hf}} + \kappa_{\text{anapole}}$$

→ Careful experiments, and good atomic many-body calculations, get:

$$\kappa_{\text{tot}}(^{133}\text{Cs}) = 0.112 \pm 0.016 \text{ (1997, Colorado)}$$

$$\kappa_{\text{tot}}(^{205}\text{Tl}) = 0.293 \pm 0.400 \text{ (1995, Seattle)}$$

$$\kappa_{\text{tot}}(^{205}\text{Tl}) = -0.08 \pm 0.40 \text{ (1995, Oxford)}$$

→ Subtraction of Z exchange and hyperfine correction, get:

$$\kappa_{\text{anapole}}(^{133}\text{Cs}) = 0.090 \pm 0.016 \text{ (7}\sigma \text{ determination)}$$

$$\kappa_{\text{anapole}}(^{205}\text{Tl}) = 0.376 \pm 0.400 \text{ (consistent with 0)}$$

## ■ Ongoing works and proposals on atomic PNC

→ Cs (Paris)

→ Tl (Seattle): sign and value?

→ Dy, Yb (Berkeley): odd neutron

→ Fr (Stony Brook, TRIUMF): isotope effect

→ NSI effect measures  $Q_W$ , NSD effect shows nuclear AM!

■ Theoretical calculations of nuclear AMs

$$\langle \Psi | T_1^{\text{el}} | \Psi \rangle = \langle \Psi_0 | T_1^{\text{el}(A)} | \Psi_0 \rangle + \sum_{\Phi_0} \left( \frac{\langle \Psi_0 | T_1^{\text{el}(V)} | \Phi_0 \rangle \langle \Phi_0 | H_{\text{PNC}}^{(2)} | \Psi_0 \rangle}{E_{\Psi_0} - E_{\Phi_0}} + H.c. \right)$$

where all states are many-body states

→ Subtleties:

1. The form of  $H_{\text{PNC}}$ : 2-body or 1-body analog

2. Current conservation: should include PNC meson exchange currents arising from  $H_{\text{PNC}}^{(2)}$ , and note that some of them might not be constrained by the gauge principle

3. Good description of nuclear structure: one of the fundamental problem in nuclear many-body physics

■ Selected results of  $\kappa_{\text{anapole}}$  using DDH best values

**A: s.p. approximation**

**B: s.p.  $\oplus$  Woods-Saxon, oscillator, full currents ...etc.**

**C: RPA with constant density approximation**

**D: RPA beyond const. density**

**E: SM with closure approximation**

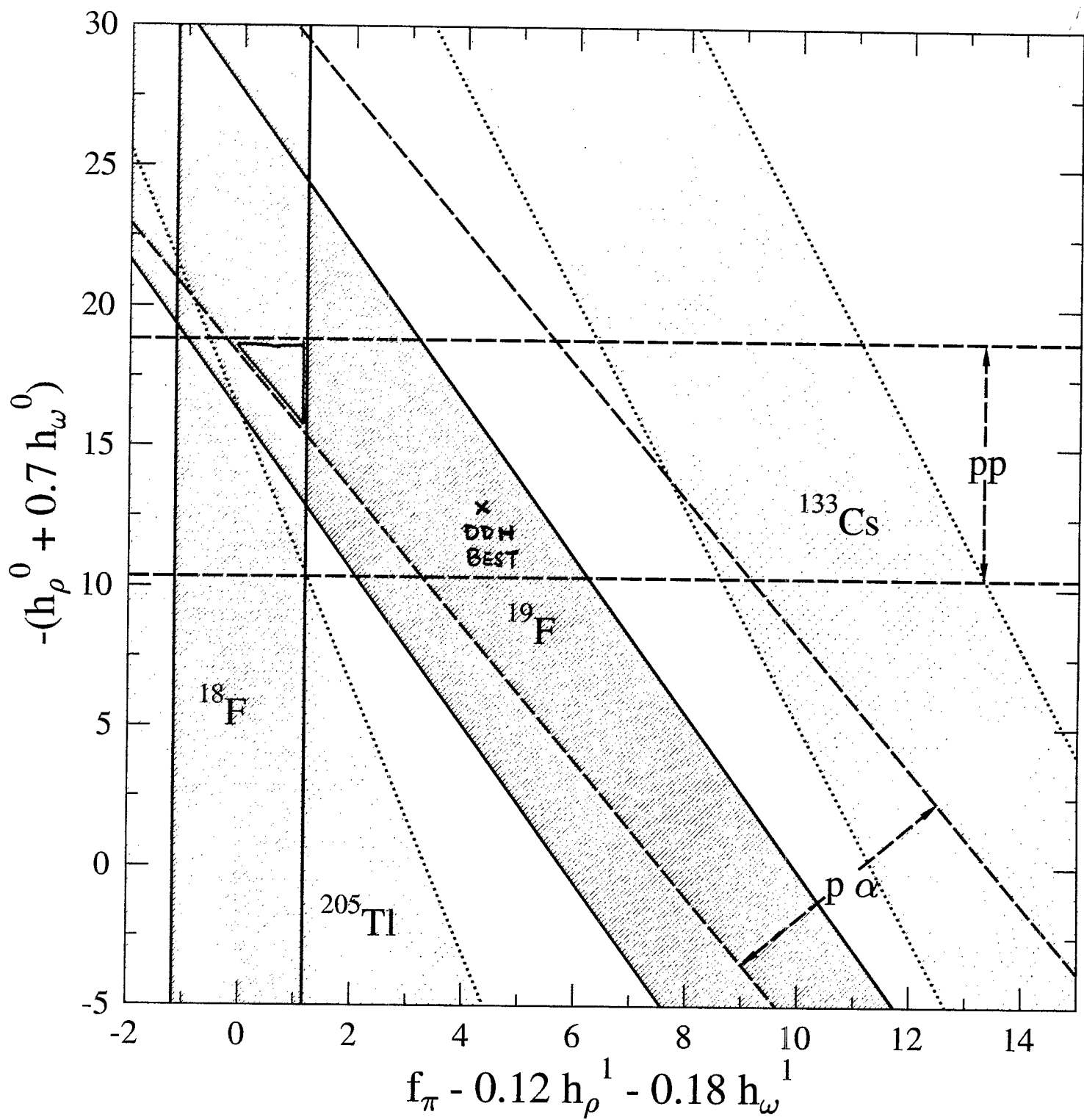
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>
$^{133}\text{Cs}$	0.09	(0.08, 0.07)	0.05	0.04	0.05
$^{205}\text{Tl}$	-0.36	(-0.32, -0.30)	-0.28	-0.18	-0.08

## ■ Detailed breakdown of various contributions to AM

Nucleus	Source	$f_\pi$	$h_\rho^0$	$h_\rho^1$	$h_\rho^2$	$h_\omega^0$	$h_\omega^1$
$^{133}\text{Cs}$	1 – body	0.59	0.87	0.90	0.36	0.28	0.29
	2 – body	8.58	0.02	0.11	0.06	-0.57	-0.57
	mixing	51.57	-16.67	-4.88	-0.06	-9.79	-4.59
	total	60.74	-15.78	-3.87	0.36	-10.09	-4.87
$^{205}\text{Tl}$	1 – body	-0.63	-0.86	-0.96	-0.35	-0.29	-0.29
	2 – body	-3.54	-0.01	-0.06	-0.03	0.28	0.28
	mixing	-13.68	4.63	1.34	0.08	2.77	1.27
	total	-18.03	3.76	0.33	-0.30	2.76	1.26

## ■ Constraints on PNC meson-nucleon couplings

Observable	Exp. ( $\times 10^{-7}$ )	$f_\pi - 0.12 h_\rho^1$ $- 0.18 h_\omega^1$	$h_\rho^0 + 0.7 h_\omega^0$	$\Delta h_\rho^1$	$h_\rho^2$	$\Delta h_\omega^0$	$\Delta h_\omega^1$
$A_Z^{\text{pp}} (13.6 \text{ MeV})$	$-0.93 \pm 0.21$		0.043	0.043	0.017	0.009	0.039
$A_Z^{\text{pp}} (45 \text{ MeV})$	$-1.57 \pm 0.23$		0.079	0.079	0.032	0.018	0.073
$A_Z^{\text{pp}} (221 \text{ MeV})$	$0.84 \pm 0.29$		-0.030	-0.030	-0.012	0.021	
$A_Z^{\text{p}\alpha} (46 \text{ MeV})$	$-3.34 \pm 0.93$	-0.340	0.140	0.006		-0.039	-0.002
$P_\gamma (^{18}\text{F}, 1081 \text{ keV})$	$1200 \pm 3860$	4385		34			-44
$A_\gamma (^{19}\text{F}, 110 \text{ keV})$	$-740 \pm 190$	-94.2	34.1	-1.1		-4.5	-0.1
$\langle    a(\text{Cs})    \rangle M_N^2 / e$	$800 \pm 140$	60.7	-15.8	3.4	0.4	1.0	6.1
$\langle    a(\text{Tl})    \rangle M_N^2 / e$	$370 \pm 390$	-18.0	3.8	-1.8	-0.3	0.1	-2.0



## ● Summary

→ Nuclear anapole moments are a sign of nuclear parity nonconservation and capable of putting good constraints on the PNC nuclear interaction. This means they provide a way to study the hadronic weak interaction and dynamics of strong interaction.

→ The theory-vs.-experiment discrepancy is one puzzle needed to be sorted out. More experiments alike are valuable, and better atomic and nuclear calculations are the challenge for theorists.